J. Fluid Mech. (1976), *vol.* 77, *part* 3, *pp.* 499–509
Printed in Great Britain

The measurement of the spectra of highly turbulent flows by a randomly triggered pulsed-wire anemometer

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(Received 21 May 1976)

The pulsed-wire anemometer has enabled velocity measurements to be made in a variety of unsteady turbulent flows. So far the use of the instrument has been confined to the determination of mean velocity, turbulence intensity and probability density. Here we show how spectral information can also be obtained. The instrument provides estimates of the velocity by measuring the time of flight of a heated flow tracer. Periodic samples of velocity can be generated by driving the anemometer with a regular train of pulses, but unfortunately it is not possible to pulse the heater at a rate greater than about 50 or so times per second, without risk of burning out the wire. This limits the spectral information that can be obtained to situations where all the energy occurs at frequencies below 25 Hz. The random sampling scheme used here avoids the aliasing problem inherent in periodic sampling and enables estimates of power spectral density to be formed up to frequencies many times the average sampling rate. This technique is used to obtain spectral estimates of the velocity fluctuations that arise at various locations in the wake of a flat plate.

1. Introduction

The measurement of flow velocity within regions of high turbulence intensity presents formidable difficulties. Conventional probe devices, such as Pitot tubes and hot-wire anemometers, which are commonly used in aerodynamic research, cannot be used in such flows with any degree of confidence because of their inability to resolve the instantaneous flow direction. Indeed, in extreme cases of fully separated flow, where reversals in direction occur, these instruments are virtually useless. If velocity information is required under these conditions it is essential to turn to other instruments which are capable of resolving the flow direction. The 'pulsed-wire anemometer' (Bradbury & Castro 1971) is one such device which has been developed precisely for making measurements of velocity in these circumstances. It has enabled useful data to be obtained in a variety of situations involving turbulent separated flows. An example of this is provided by measurements made in the wake of bluff bodies (Bradbury 1976). This instrument provides a measure of the flow velocity by recording the flight time of a tracer of heated air which is introduced into the flow by an electrically pulsed heater wire.

The time the heated zone of fluid takes to reach a detector provides a measure of the flow velocity at that instant. The heater wire has a diameter of 9 μm and a length of 10 mm, and so actually a cylindrical zone of heated fluid is created. The hot fluid is convected downstream, where it passes over a fine resistance thermometer that acts as the detector. The resistance thermometer's element has its axis at right angles to the heater wire so that tracers can be detected over a very wide cone of flow directions. Detector elements were mounted both upstream and downstream of the heater so that velocities of either sign could be measured. With this arrangement, meaningful measurements of mean velocity, turbulence intensity and probability density could be obtained, even in extremely turbulent flows.

So far no attempt has been made to obtain time-domain information from this instrument. The pulsed-wire instrument can provide sampled values of the continuously varying velocity at only discrete instants, and we are concerned here with the problem of extracting the power spectrum of the velocity fluctuations from such data. A periodic sequence of pulses fed to the heater element would provide sampled values of the velocity at equally spaced instants, and these could be used to compute spectral estimates, either by the Blackman–Tukey correlation method or by the rather faster direct 'Fast Fourier Transform'. Methods based on periodically sampled arrays enable spectral estimates to be formed at frequencies up to one-half of the sampling rate; but a difficulty arises in that any energy contained in the signal above this so-called 'Nyquist' frequency is folded down and aliases the low frequency estimates. This does not present any problem in the more usual case of a continuous signal, where an analog low-pass filter would normally be employed to remove all the high frequency components prior to digitization, but it does cause difficulties in the present situation, where the data arise discretely and cannot be pre-filtered. Periodic sampling can only be used if the sampling rate is fast enough to resolve completely all the high frequency components of the flow fluctuations. Unfortunately there are practical limitations on the rate at which pulses can be fired, and this restricts the use of periodic sampling to flows that only contain fluctuations with frequencies below about 20 or 30 Hz. This limit is set by the maximum pulsing rate that can be applied continuously to the heater wire without its burning out. It turns out that the restricted frequency range of the spectral estimates generated from periodically sampled data and the associated problem of aliasing can be avoided by employing a random sampling scheme (Gaster & Roberts 1975). Valid alias-free estimates can be formed at any desired frequency from a Poisson-sampled signal. There is, of course, a price to pay for circumventing aliasing in this way, and this takes the form of increased variability of the spectral estimates over and above that expected from a signal that is periodically sampled. Spectral estimation from randomly acquired data has been considered mainly in situations where the physical process involved in creating the samples is naturally random, but it appears that these ideas can also be exploited to advantage in other cases, such as the present one, where the sampling can be prescribed.

Here we show how spectra of the velocity fluctuations in highly turbulent flows can be obtained from data provided by a randomly triggered pulsed-wire anemometer. It should be stressed, however, that these experiments were carried out primarily to demonstrate the feasibility of the method, and are therefore of only a preliminary nature. As the results show, there are difficulties in obtaining values of the autocorrelation at very short lag times, because of various practical considerations with the present pulsed-wire anemometer, and further work is required to minimize this loss of information.

2. Instrumentation

In the usual mode of operation the pulsed-wire anemometer is triggered at regular time intervals, and on each occasion the time of flight of the heated fluid tracer is registered on a four-digit counter preceded by a sign character. The sign character indicates which of the two sensor wires has captured the tracer, and reveals the flow direction. This information is generally fed into a computer system together with the necessary calibration data, so that the mean velocity, turbulence intensity and probability density can be calculated. When time-domain information is to be extracted from the data it is also necessary to record the instant at which each pulse was triggered. These times were recorded to an accuracy of 1 ms on a continuously running four-digit counter fed with 1 kHz clock pulses. Each data sample consisted of a sign character, followed by the three most significant digits of the time-of-flight counter, followed by four digits from the running clock. These eight characters were assembled in a serial form and stored in a buffer store, which was coupled to the NPL packet switching network. This network enabled the digital information stored in the buffer to be transmitted to a large file store in packets of 128 eight-bit bytes. Each sample from the pulsed-wire instrument consisted of 8 characters, 512 of which filled the 4096 locations in the buffer. When the buffer became full the control lines on the interface initiated the transfer of data through the network to the file store, and, when this was complete, the appropriate control lines automatically restarted the cycle. The time taken to fill the buffer depended on the sampling rate and took between 15 and 20 s, whilst the transmission of data occupied only about 1 s.

The random pulse train which was used to trigger the anemometer was derived from a Gaussian white-noise source. A pulse was generated every time this noise signal exceeded some pre-set threshold level. This process can be expected to generate a train of pulses with Poisson-distributed arrival times. The average pulse rate was controlled by adjustment of the amplitude of the noise signal relative to the trigger level of the pulse generator. Ideally, a Poisson sequence would contain some intervals between neighbouring pulses which were very short, but, since it is not possible to drive the pulsed-wire anemometer with such a sequence, as it is necessary to allow sufficient time to elapse for the electronic counting circuits to reset before initiating any further cycles, some means had to be provided to prevent such an occurrence. A monostable with an adjustable time constant was therefore introduced between the pulse generator and the anemometer unit to exclude those pulses which arrived after an insufficient interval. The consequences of introducing this 'dead-zone' will be brought out in the discussion of the results.

Run		$\frac{x}{b}$	$\frac{y}{b}$	$\frac{\text{Mean velocity}}{\text{Free-stream velocity}}$	$\frac{\text{Intensity}}{\text{Free-stream velocity}}$
A	u component	2	0	$-0{\cdot}03$	0·36
B	v component	2	0	0	0·64
C	u component	3	0	$+0{\cdot}39$	0·27
D	u component	1	1	$+1{\cdot}17$	0·23

TABLE 1. Measurement locations. $b = 2$ in., free-stream velocity $U_0 = 17{\cdot}9$ ft/s.

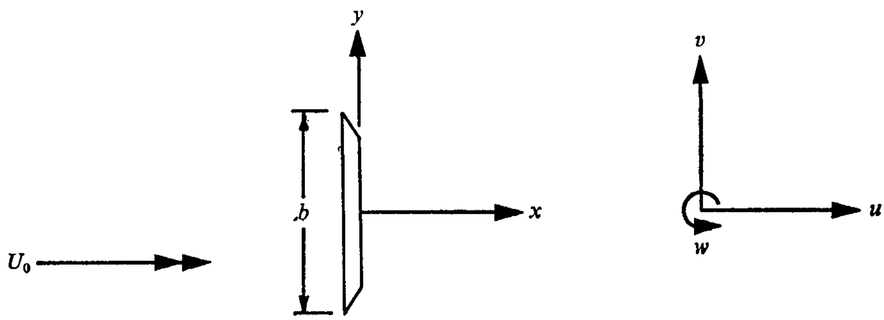

FIGURE 1. Notation for table 1.

3. Experimental arrangement

The experiments were carried out in the 3×3 ft closed-return wind tunnel at the National Physical Laboratory. The measurements to be reported were made in the separated wake of a two-dimensional flat plate mounted normal to the free stream. The plate, which spanned the working section of the tunnel, was fitted with end plates in order to reduce the influence of the side-wall boundary layers on the overall structure of the flow. This model has been previously used by Bradbury (1976) in a study of the velocities within the separated wake in which a pulsed-wire anemometer was used.

The spectra of the velocity fluctuations are presented for three locations behind this plate. These positions are given in table 1 together with the mean velocities and turbulence intensities measured at these locations. Figure 1 defines the axes and symbols used. In examples A, B and C the time-of-flight counter on the anemometer was run from a 1 MHz clock, the 'dead' period was set at 11 ms. and the mean sampling rate was adjusted to a value of roughly 30 samples/s. In example D the clock rate was increased to 2 MHz and the dead period reduced to 5 ms. For this run the sampling rate was increased to 50 samples/s.

4. Data processing

The data generated by the anemometer and the time when the sample was created were written into a buffer store and then transmitted via the network to the file store. Because there was a maximum length of file that could be written

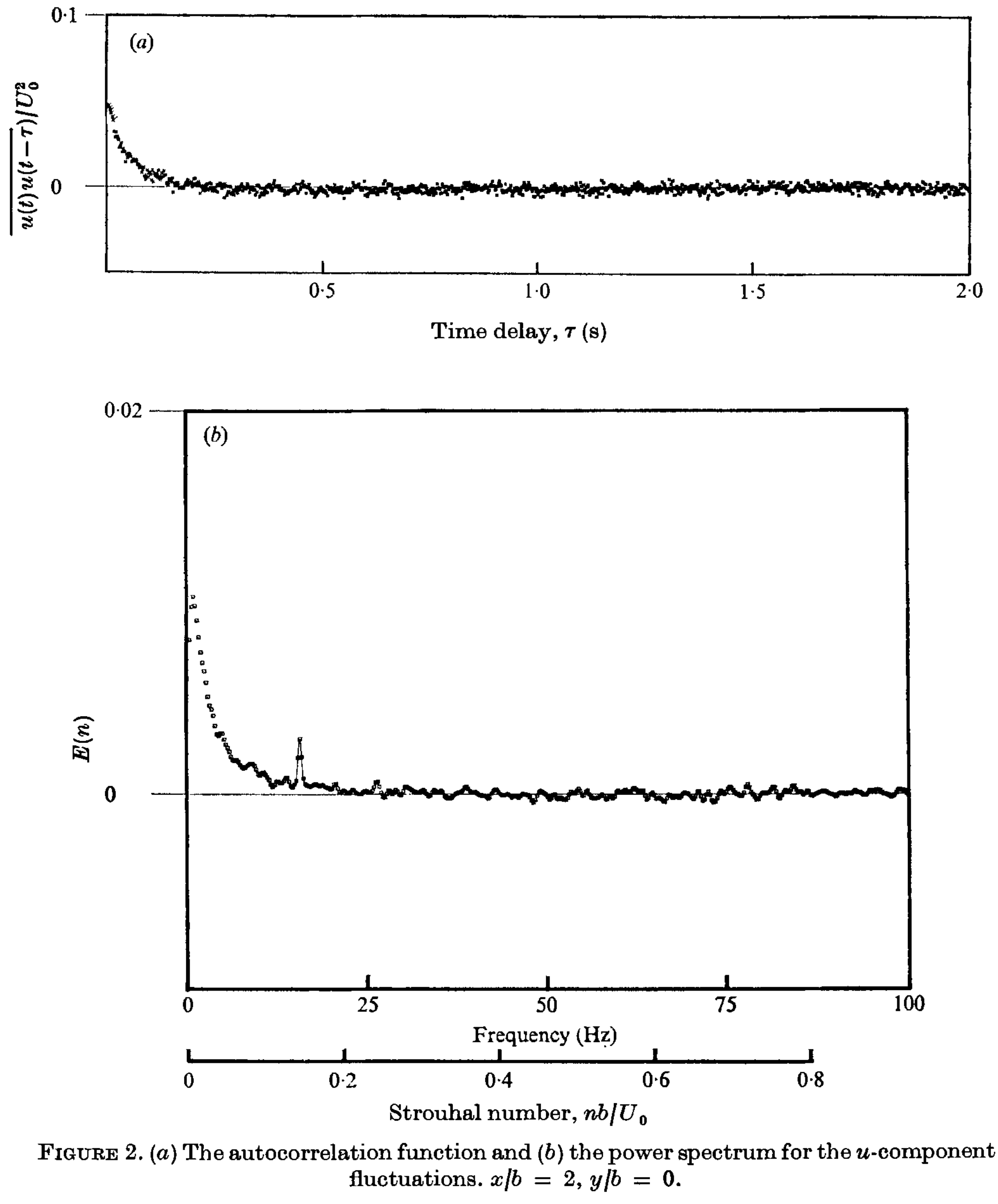

FIGURE 2. (a) The autocorrelation function and (b) the power spectrum for the u-component fluctuations. $x/b = 2$, $y/b = 0$.

onto the file store at one time it was necessary to limit each file to a maximum size of 30 buffer blocks. For each example six separate files of this length were created, and these provided a total of 180 blocks of 4096, or 737 280 characters. This represented 92 160 samples at each location. The file-store records were transferred to KDF9 via the network link and then permanently stored on magnetic tape. Any data samples which failed to conform to the input format of a sign character followed by 7 numeric characters were ignored. There were typically 20 or 30 of these spurious samples in each set of 90 000 samples. The data stored on tape were used to compute the autocorrelation function by the slotting method discussed by Gaster & Roberts (1975). Each time of flight was converted to a velocity, using the calibration data of the probe, and intervals between samples were then used to decide within which slot the cross-product should be included. The running clock had a resolution of 1 ms and this interval was conveniently used as a half-slot width in the correlation plane. A maximum lag time of 2 s was chosen, and this interval was divided into a thousand slots. The mean cross-products of sample pairs falling within each slot were then transformed to produce power spectral estimates. In the first three examples typically some 5000 cross-products appeared in each slot, whilst at the higher sampling rate used in the last example this rose to about 10000.

5. Discussion of results

Possibly the most interesting results arise from data sets A and B, which concern the u (streamwise) and v (normal) components of velocity at the position in the recirculating flow behind the plate where the mean values of both components are virtually zero. The autocorrelation functions for these are shown on figures 2 (a) and 3 (a) respectively. Even though there is some scatter, consistent mean trends can be identified. The autocorrelation of the streamwise component, plotted on figure 2 (a), has a rapid decay followed by a slight overshoot of the time axis, while the normal component, shown on figure 3 (a), exhibits a well-defined decaying sinusoid.

No information was available for the autocorrelations at the first six locations, associated with lag times less than 10 ms, because these were excluded from the data for the practical reasons discussed in §3. However, in order to obtain the power spectral densities from the Fourier cosine transform of the autocorrelation curves, it was necessary to assign plausible values to the missing data points before proceeding. Suitable values were estimated by sketching in the missing portion of the curve with a smooth line in such a way as to comply with the following constraints: the line had to cross the ordinate normally at a level higher than the maximum modulus of the known part of the correlation function, but below that of the signal's mean square. Any high frequency motion present in the flow, or any spurious noise for that matter, can be expected to contribute a narrow spike to the correlation function at zero time lag. Although the peak value of this spike is given by the mean square of the complete signal, which is known, this does not provide a good guide to the quantity required, which is the average value of the correlation over a slot width at zero lag. The values obtained by the above process are shown as flagged symbols on the figures. It is clear that errors made in estimating the initial correlation values over a lag time of Δt, say, will be reflected in the spectrum by the addition of the cosine transform of this error. For $\omega \Delta t < 1$ this results in a bias to the spectrum proportional to Δt. At high frequency the bias decays in a manner that depends on details of the error within Δt. The following results must be interpreted with this in mind.

The power spectral estimates shown on figures 2 (b) and 3 (b) are the cosine transforms of figures 2 (a) and 3 (a). It should be noted that the scale of the v-com-

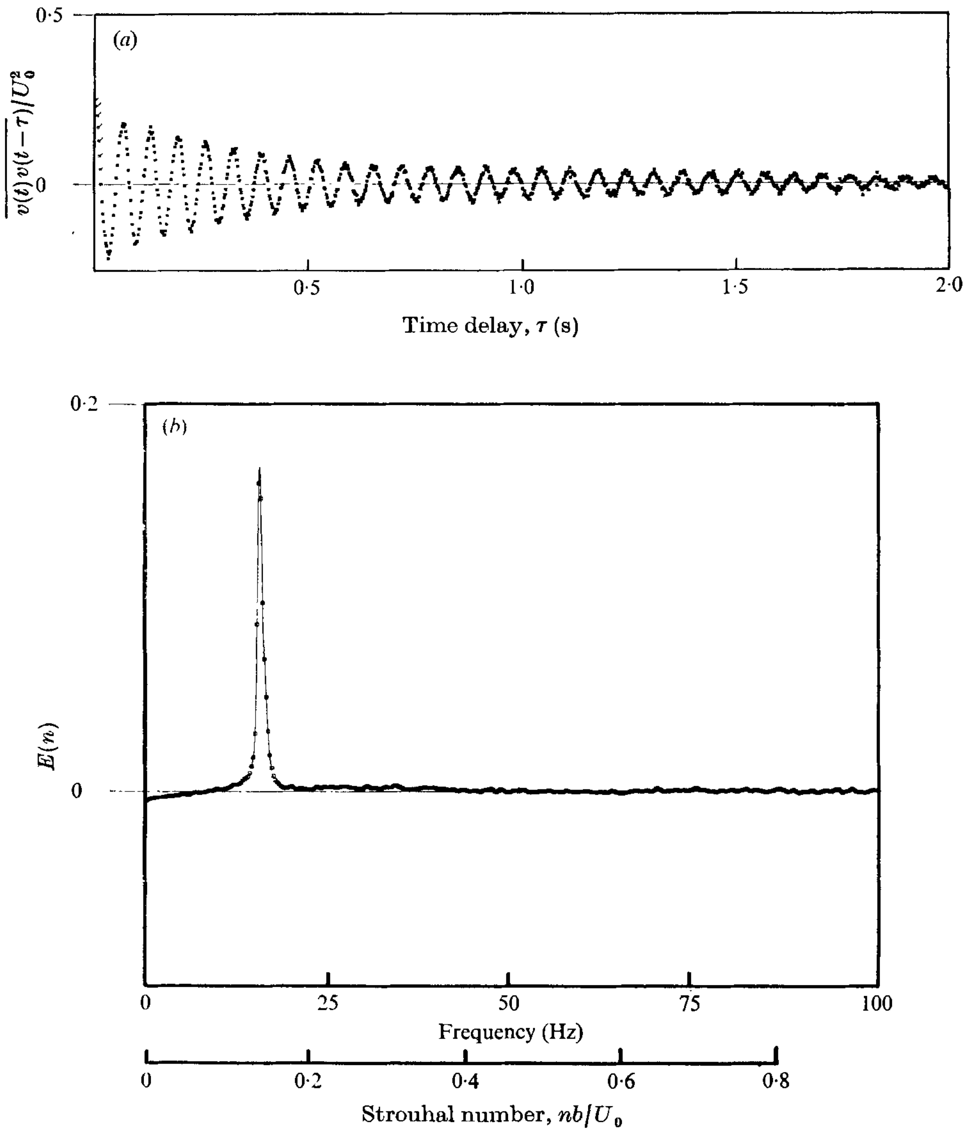

FIGURE 3. (*a*) The autocorrelation function and (*b*) the power spectrum for the v-component fluctuations. $x/b = 2$, $y/b = 0$.

ponent spectrum is 10 times that of the u-component spectrum. It is apparent that the v fluctuations arise mainly from the highly periodic vortex-shedding motion in the wake, and that much of their energy is contained in a sharp peak at a Strouhal number nd/U of 0·14. This shedding frequency agrees closely with values found previously from signals obtained from hot wires positioned outside the wake of the flat plate, where such instruments can be used with confidence. The negative spectral estimates that appear at low frequencies almost certainly arise from incorrect assignment of the initial six autocorrelation values and this example serves as an illustration of the difficulty associated with the 'dead-zone'.

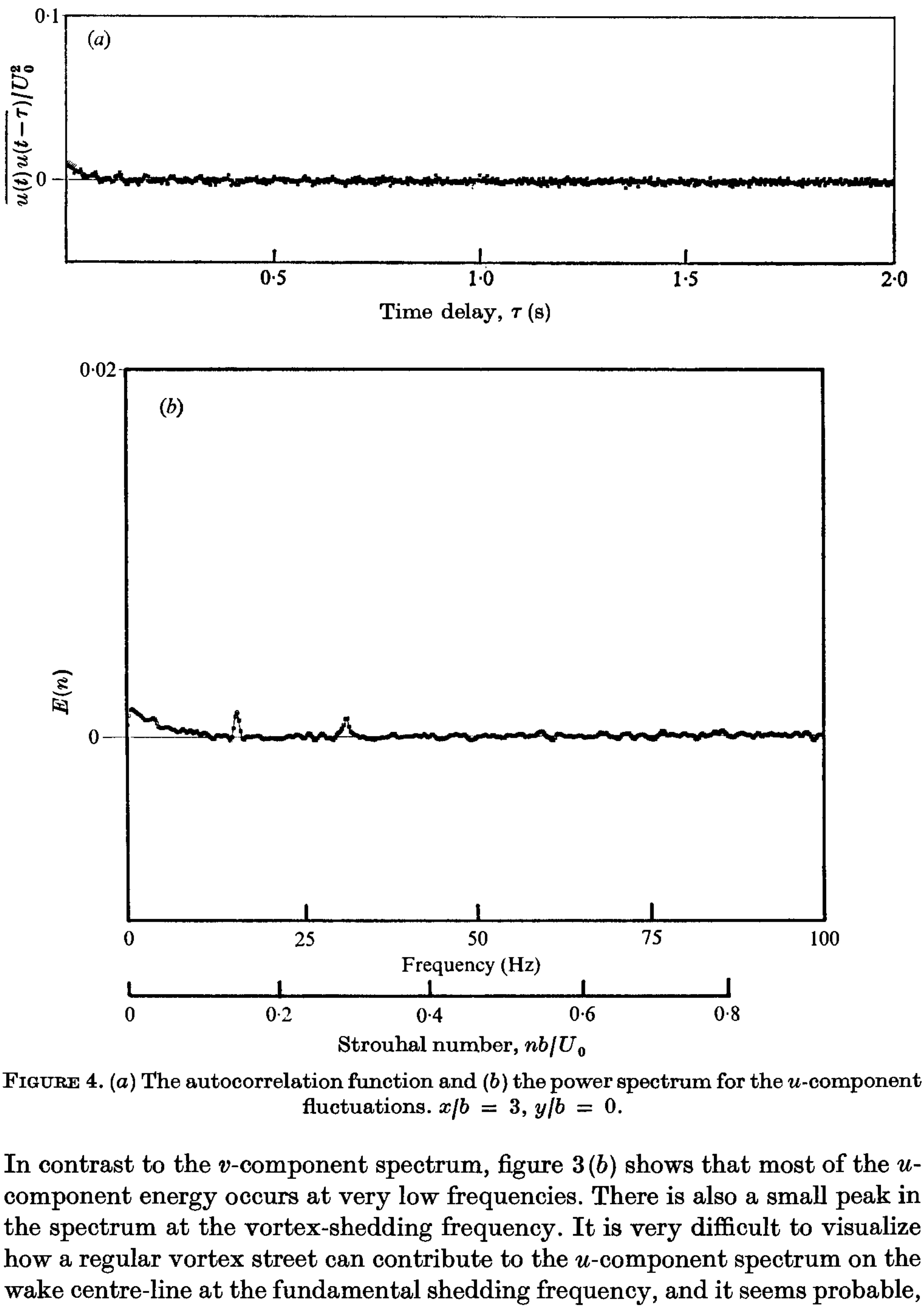

FIGURE 4. (a) The autocorrelation function and (b) the power spectrum for the u-component fluctuations. $x/b = 3$, $y/b = 0$.

In contrast to the v-component spectrum, figure 3(b) shows that most of the u-component energy occurs at very low frequencies. There is also a small peak in the spectrum at the vortex-shedding frequency. It is very difficult to visualize how a regular vortex street can contribute to the u-component spectrum on the wake centre-line at the fundamental shedding frequency, and it seems probable, therefore, that this peak arises from imperfections in the yaw response of this instrument. It has been shown (Bradbury 1976) that the effective u-component of velocity u_e measured by a pulsed-wire anemometer can be empirically represented by the expression

$$u_e = u + \epsilon v_r,$$

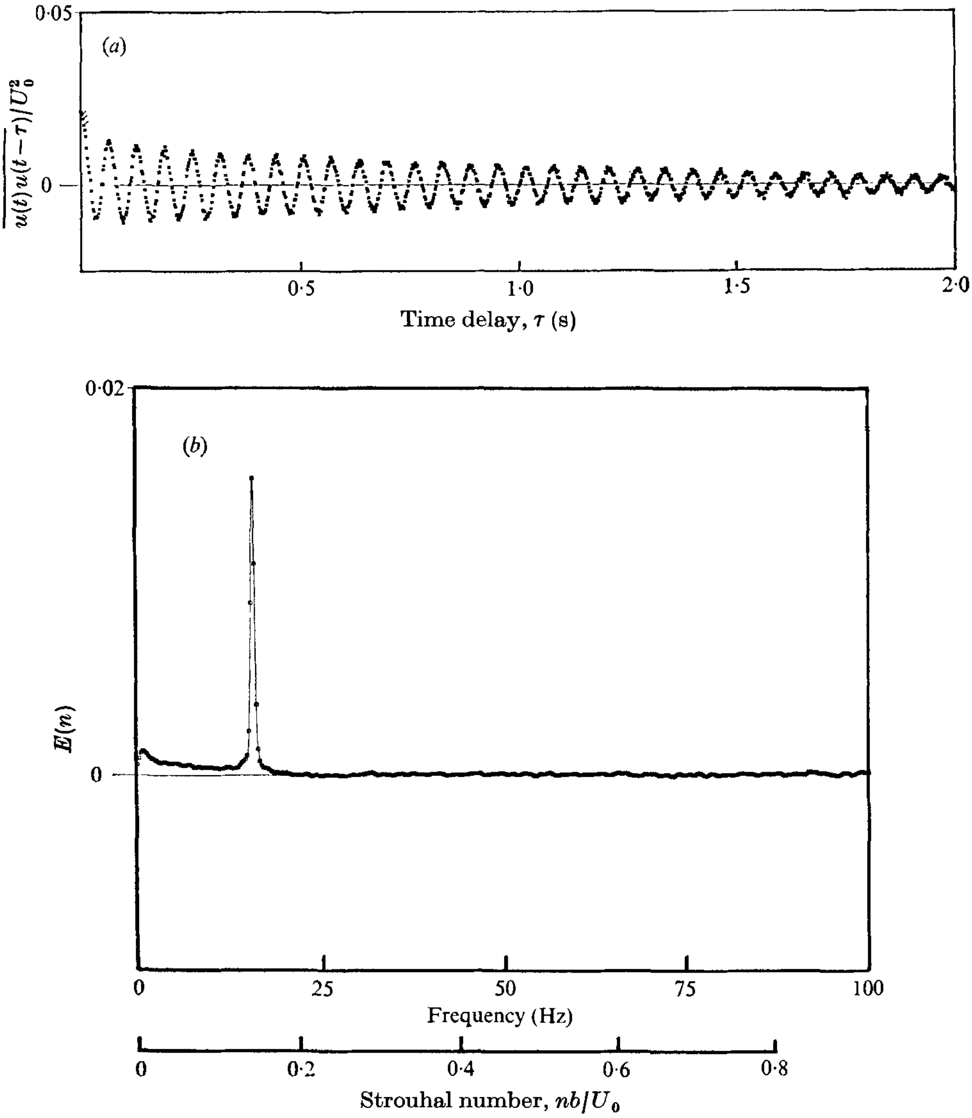

FIGURE 5. (a) The autocorrelation function and (b) the power spectrum for the u-component fluctuations. $x/b = 1$, $y/b = 1$.

where $v_r = [v^2 + w^2]^{\frac{1}{2}}$ is the radial velocity normal to the u component and ϵ is a constant with a value of about 0·1. On the flow centre-line, u and v_r will be uncorrelated, so that the measured autocorrelation function will be given by

$$\overline{u_e(t)\,u_e(t-\tau)} = \overline{u(t)\,u(t-\tau)} + \epsilon^2\overline{v_r(t)\,v_r(t-\tau)}.$$

Here $(\overline{v^2})^{\frac{1}{2}}$ is approximately twice $(\overline{u^2})^{\frac{1}{2}}$ and $(\overline{w^2})^{\frac{1}{2}}$, so that the energy spectrum of the u-component fluctuations, as given by this device, will contain about 1 % of the v-component energy and vice versa. Normally this would not be significant, but here the high v-component fluctuations are concentrated in a narrow frequency band, and this 'cross-talk' is almost certainly responsible for the small peak in the measured u-component spectrum.

Figures 4 (a) and (b) show the autocorrelation and spectrum respectively of the u-component velocity fluctuations on the centre-line of the wake three plate widths downstream, on the same scale as figures 3 (a) and (b). A second harmonic peak is just discernible in the spectrum, and whilst this is almost certainly genuine the peak at the fundamental again probably arises from the imperfect yaw response of the probe.

Some of the consequences of being unable to determine the correlation at short lag times has been demonstrated in the foregoing examples. In the last set of data to be shown, attempts were made to reduce the 'dead-zone' to one-half of its previous value. This involved alterations to the internal clock and counting circuits in the anemometer, and resulted in the loss of only the first three correlation values. Figure 5 (a) shows the correlation function obtained, and it is clear that there is little difficulty in this case in assigning reasonable values to the missing data. The resulting spectrum of figure 5 (b) can therefore be presented with greater confidence than those previously shown. The periodic nature of the flow is particularly well defined and indicates a strong shedding motion at this location together with a broad-band contribution to the low frequency part of the spectrum.

6. Conclusions

A randomly fired pulsed-wire anemometer can provide the data necessary for the determination of spectra. It has been shown how spectral estimates can be formed at frequencies up to one or two hundred hertz, even though sampling takes place at an average rate of only 50 or so times per second.

The loss of correlation information at short lag times does, however, influence the shape of the resulting power spectrum, and it is worth making some effort to reduce this period as much as possible. This can be done by reducing the time constant of the pulsed wire, through the use of finer wire, and by making the necessary alterations to the electronic timing circuits. By these means it is hoped to be able to reduce the 'dead-zone' to two milliseconds or less. It is possible to define upper and lower limits on the values of the missing correlation coefficients outside which they cannot lie without violating a fundamental theorem. This states that the Toeplitz matrix formed from the correlation coefficients must be semi-positive definite, a condition that ensures that all the spectral estimates are positive. This inequality has been used by Burg (1967) to extend correlation functions to longer lags in a plausible way (maximum entropy spectral analysis), but it can also be used to provide bounds on the initial correlation coefficients.

Additional variability is introduced by employing a random sampling scheme and a large amount of data has therefore to be processed in order to achieve estimates of sufficient quality. However, there is every prospect that with the present developments in microprocessors a dedicated computer system could be used to batch process the data as it is acquired, and this would obviate the need for the large storage facilities which were necessary here. The direct transform approach to spectral estimation, which needs to consider only short blocks of data at a time, can be used to good effect in these circumstances (Gaster & Roberts 1977).

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